CLAIMS

What is claimed is:

- 1. A method for configuring an equivalent $2^n \times 2^n$ k-stage bit-permuting network
- 5 based on a given $2^n \times 2^n$ k-stage bit-permuting network having the representation

 $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{k-1}:\sigma_k]_n,$ the method comprising

specifying a permutation κ on integers from 1 to n that preserves n, and implementing the equivalent network as $[\sigma_0:\sigma_1:\ldots:\sigma_{j-1}\kappa:\kappa^{-1}\sigma_j:\ldots:$

 σ_k]_n, j = 1, 2, ..., or k.

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- 2. The method as recited in claim 1 wherein the given network is a banyan-type network and the equivalent network is a banyan-type network.
 - 3. A method for configuring an equivalent $2^n \times 2^n$ k-stage bit-permuting network
- based on a given $2^n \times 2^n$ k-stage bit-permuting network having the representation

 $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{k-1}:\sigma_k]_n,$ the method comprising

specifying permutations $\kappa_1,\,\kappa_2,\,\ldots\,,\,\kappa_k$ on integers from 1 to n that preserve

n, and

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implementing the equivalent network as $[\sigma_0\kappa_1:\kappa_1^{-1}\sigma_1\kappa_2:\kappa_2^{-1}\sigma_2\kappa_3:\dots:$ $\kappa_{k-1}^{-1}\sigma_{k-1}\kappa_k:\kappa_k^{-1}\sigma_k]_n.$

- 4. The method as recited in claim 3 wherein the given network is a banyan-type
- 5 network and the equivalent network is a banyan-type network.

succeeding exchange, respectively, and

- 5. A method for configuring an equivalent $2^n \times 2^n$ bit-permuting network based on a given $2^n \times 2^n$ bit-permuting network composed of stages and exchanges, the method comprising
- identifying one stage from the stages of the given network, the identified stage having a preceding exchange immediately before it and a succeeding exchange immediately after it,

specifying a permutation on the integers 1 to n that preserves n,

rearranging the preceding exchange and the succeeding exchange with reference to the permutation to generate a rearranged preceding exchange and a rearranged

implementing the equivalent network so that a stage in the equivalent network corresponding to the identified stage has the rearranged preceding exchange and

the rearranged succeeding exchange.

- 6. The method as recited in claim 5 wherein the permutation, denoted as κ, induces a 2ⁿ×2ⁿ cell rearrangement exchange X_κ, and the rearranging includes multiplying the
 5 preceding exchange by X_κ from the right-hand side to produce the rearranged preceding exchange and multiplying the succeeding exchange by X_κ⁻¹ from the left-hand side to produce the rearranged succeeding exchange.
 - 7. The method as recited in claim 6 wherein the given network has k-stages, the given network has the representation $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{k-1}:\sigma_k]_n$, the identified stage is stage j, and the equivalent network is of the form $[\sigma_0:\sigma_1:\ldots:\sigma_{j-1}\kappa:\kappa^{-1}\sigma_j:\ldots:\sigma_k]_n$, $j=1,2,\ldots,$ or k.
- 8. The method as recited in claim 5 wherein the given network is a banyan-type network and the equivalent network is a banyan-type network.
 - 9. A method for configuring an equivalent $2^n \times 2^n$ bit-permuting network by cell rearrangement based on a given $2^n \times 2^n$ bit-permuting network composed of stages and

exchanges, the method comprising

identifying one stage from the stages of the given network, the identified stage having a preceding exchange and a succeeding exchange,

specifying a permutation, denoted as κ , on the integers 1 to n that preserves $\mbox{\bf n} \mbox{ and induces a } 2^n \times 2^n \mbox{ cell rearrangement exchange } X_{\kappa},$

rearranging the preceding exchange by multiplying the preceding exchange with X_{κ} from the right-hand side to produce a rearranged preceding exchange and by multiplying the succeeding exchange by X_{κ}^{-1} from the left-hand side to produce a rearranged succeeding exchange, and

implementing the equivalent network so that a stage in the equivalent network corresponding to the identified stage has the rearranged preceding exchange and the rearranged succeeding exchange.

10. A method for cell rearrangement of a 2ⁿ×2ⁿ bit-permuting network composed of
 stages and exchanges, the method comprising

selecting one stage from the stages of the given network to identify a preceding exchange and a succeeding exchange,

specifying a permutation, denoted as K, on the integers 1 to n that preserves

n and induces a $2^n \times 2^n$ cell rearrangement exchange X_K , and

multiplying the preceding exchange with X_{κ} from the right-hand side to implement a rearranged preceding exchange and multiplying the succeeding exchange by X_{κ}^{-1} from the left-hand side to implement a rearranged succeeding exchange.

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11. A method for cell rearrangement of a given stage of a 2ⁿ×2ⁿ bit-permuting network composed of stages and exchanges, the method comprising

specifying a permutation, denoted as κ , on the integers 1 to n that preserves n and induces a $2^n\times 2^n$ cell rearrangement exchange X_{κ}

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multiplying the preceding exchange immediately before the given stage by X_{κ} from the right-hand side to implement a rearranged preceding exchange for the given stage and multiplying the succeeding exchange immediately after the given stage exchange by X_{κ}^{-1} from the left-hand side to implement a rearranged succeeding exchange for the given stage.

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12. A method for rearranging a given $2^n \times 2^n$ bit-permuting network having the representation $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{k-1}:\sigma_k]_n$ to an equivalent $2^n \times 2^n$ bit-permuting network having the representation $[\pi_0:\pi_1:\pi_2:\ldots:\pi_{k-1}:\pi_k]_n$, the method comprising

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determining permutations $\kappa_1,\,\kappa_2,\,\dots\,,\,\kappa_k$ on integers from 1 to n that preserve n, and

implementing the equivalent network with exchanges determined from $\pi_1 = \kappa_1^{-1} \sigma_1 \kappa_2$, $\pi_2 = \kappa_2^{-1} \sigma_2 \kappa_3$, ..., $\pi_{k-1} = \kappa_{k-1}^{-1} \sigma_{k-1} \kappa_k$ so that the equivalent network can be further expressed as $[\alpha : \kappa_1^{-1} \sigma_1 \kappa_2 : \kappa_2^{-1} \sigma_2 \kappa_3 : ... : \kappa_{k-1}^{-1} \sigma_{k-1} \kappa_k : \beta]_n$ for arbitrary permutations α and β .

- 13. The method as recited in claim 12 wherein an input exchange $\alpha = \pi_0$ is prepended to the equivalent network.
- 14. The method as recited in claim 12 wherein an output exchange $\beta = \pi_k$ is appended to the equivalent network.
- 15. The method as recited in claim 12 wherein an input exchange $\alpha=\pi_0$ is prepended to the equivalent network and an output exchange $\beta=\pi_k$ is appended to the equivalent network.
 - 16. A method for configuring a given $2^n \times 2^n$ k-stage bit-permuting network to

achieve a desired trace, the method comprising

determining a permutation σ on the integers 1 to n that maps the trace of the given network term-by-term to the desired trace, and

prepending the given network with an extra input exchange induced by σ^{-1}

- 5 if the permutation σ exists.
 - 17. A method as recited in claim 16 wherein k = n and the bit-permuting network is a $2^n \times 2^n$ banyan-type network.
- 18. A method as recited in claim 16 wherein the trace of the given network is the sequence $t_1, t_2, ..., t_k$, the desired trace is the sequence $t'_1, t'_2, ..., t'_k$, and $t'_j = \sigma(t_j)$ for j = 1, 2, ..., k.
- 19. A method for configuring a given 2ⁿ×2ⁿ k-stage bit-permuting network to
 achieve a desired guide, the method comprising

determining a permutation π on the integers 1 to n that maps the guide of the given network term-by-term to the desired guide, and

appending the given network with an extra output exchange induced by $\boldsymbol{\pi}$ if

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the permutation π exists.

20. A method as recited in claim 19 wherein k = n and the bit-permuting network is a $2^n \times 2^n$ banyan-type network.

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- 21. A method as recited in claim 19 wherein the guide of the given network is the sequence $g_1, g_2, ..., g_k$, the desired guide is the sequence $g'_1, g'_2, ..., g'_k$, and $g'_j = \pi(g_j)$ for j = 1, 2, ..., k.
- 22. A method for configuring a given $2^n \times 2^n$ k-stage bit-permuting network to achieve a desired trace and a desired guide, the method comprising

determining a permutation σ on the integers 1 to n that maps the trace of the given network term-by-term to the desired trace,

determining a permutation π on the integers 1 to n that maps the guide of the given network term-by-term to the desired guide, and

if both the permutations σ and π exist, prepending the given network with an extra input exchange induced by σ^{-1} , and appending the given network with an extra output exchange induced by π .

- 23. A method as recited in claim 22 wherein k = n and the bit-permuting network is a $2^n \times 2^n$ banyan-type network.
- 24. A method as recited in claim 22 wherein the trace of the given network is the
 5 sequence t₁, t₂, ..., t_k, the desired trace is the sequence t'₁, t'₂, ..., t'_k, and t'_j = σ(t_j) for j = 1,
 2, ..., k and wherein the guide of the given network is the sequence g₁, g₂, ..., g_k, the
 desired guide is the sequence g'₁, g'₂, ..., g'_k, and g'_j = π(g_j) for j = 1, 2, ..., k.
 - 25. A method for rearranging a given $2^n \times 2^n$ banyan-type network having the representation $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{n-1}:\sigma_n]_n$ to an equivalent $2^n \times 2^n$ banyan-type network having the representation $[\pi_0:\pi_1:\pi_2:\ldots:\pi_{n-1}:\pi_n]_n$, the method comprising determining permutations $\kappa_1,\,\kappa_2,\,\ldots,\,\kappa_n$ on integers from 1 to n that preserve n, and

implementing the equivalent network with exchanges determined from $\pi_1 = \kappa_1^{-1} \sigma_1 \kappa_2, \, \pi_2 = \kappa_2^{-1} \sigma_2 \kappa_3, \, \dots, \, \pi_{n-1} = \kappa_{n-1}^{-1} \sigma_{n-1} \kappa_n \text{ so that the equivalent network can be}$ further expressed as $[\alpha:\kappa_1^{-1} \sigma_1 \kappa_2:\kappa_2^{-1} \sigma_2 \kappa_3:\dots:\kappa_{n-1}^{-1} \sigma_{n-1} \kappa_n:\beta]_n$ for arbitrary permutations α and β .

- 26. The method as recited in claim 25 wherein an input exchange $\alpha=\pi_0$ is prepended to the equivalent network.
- 27. The method as recited in claim 25 wherein an output exchange $\beta = \pi_n$ is appended to the equivalent network.
 - 28. The method as recited in claim 25 wherein an input exchange $\alpha=\pi_0$ is prepended to the equivalent network and an output exchange $\beta=\pi_n$ is appended to the equivalent network.

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29. A method for rearranging a first $2^n \times 2^n$ banyan-type network having the representation $[\sigma_0:\sigma_1:\ldots:\sigma_{n-1}:\sigma_n]$ with a first trace induced by a permutation τ on integers 1 to n and a first guide induced by a permutation γ on integers 1 to n to a second $2^n \times 2^n$ banyan-type network having the representation $[\lambda \sigma_0:\sigma_1:\ldots:\sigma_{n-1}:\sigma_n\pi]$, the method comprising

prepending an additional input exchange X_λ to the first network, and appending an additional output exchange X_π to the first network, wherein the second network in characterized by a second trace induced by a permutation τ' on

integers 1 to n and a second guide induced by a permutation γ' on integers 1 to n such that τ' $= \lambda^{-1}\tau \text{ and } \gamma' = \pi\gamma.$

- 30. The method as recited in claim 29 wherein the permutations τ and γ that induce the first trace and the first guide are converted to any τ' and γ' , respectively, with the prepended input exchange X_{λ} and the appended output exchange X_{π} by computing $\lambda = \tau'^{-1}\tau \text{ and } \pi = \gamma^{-1}\gamma'.$
- 31. A method for configuring a given 2ⁿ×2ⁿ banyan-type network to achieve a
 10 desired trace wherein the trace of the given network is induced by a permutation τ on integers 1 to n, and the desired trace is induced by another permutation τ' on integers 1 to n, the method comprising

determining a permutation $\lambda = \tau'^{-1}\tau$, and prepending the given network with an extra input exchange induced by λ .

32. A method as recited in claim 31 wherein the desired trace is 1, 2, ..., n and the permutation $\lambda = \tau$.

- 33. A method as recited in claim 31 wherein the desired trace is n, n-1, ..., 1 and the permutation $\lambda = \sigma_{\leftrightarrow}^{(n)} \tau$.
- 34. A method for configuring a given 2ⁿ×2ⁿ banyan-type network to achieve a
 5 desired guide wherein the guide of the given network is induced by a permutation γ on integers 1 to n, and the desired guide is induced by another permutation γ' on integers 1 to n, the method comprising

determining a permutation $\pi = \gamma^{-1} \gamma'$, and appending the given network with an extra output exchange induced by π .

- 35. A method as recited in claim 34 wherein the desired guide is 1, 2, ..., n and the permutation $\pi = \gamma^{-1}$.
- 36. A method as recited in claim 34 wherein the desired guide is n, n-1, ..., 1 and the permutation $\pi = \gamma^{-1} \sigma_{\leftrightarrow}^{(n)}$.
 - 37. A method for configuring a given 2ⁿ×2ⁿ banyan-type network to achieve a desired trace and a desired guide wherein the trace of the given network is induced by a

permutation τ on integers 1 to n, the desired trace is induced by another permutation τ' on integers 1 to n, the guide of the given network is induced by a permutation γ on integers 1 to n, and the desired guide is induced by another permutation γ' on integers 1 to n, the method comprising

5 determining a permutation $\lambda = \tau'^{-1}\tau$,

determining a permutation $\pi = \gamma^{-1} \gamma'$,

prepending the given network with an extra input exchange induced by λ ,

and

appending the given network with an extra output exchange induced by π .

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38. An equivalent $2^n \times 2^n$ k-stage bit-permuting network based on a given $2^n \times 2^n$ k-stage bit-permuting network having the representation $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{k-1}:\sigma_k]_n$, the equivalent network comprising

permutation means for computing a permutation κ on integers from 1 to n

15 that preserves n, and

a $2^n\times 2^n$ k-stage bit-permuting network configured as $[\sigma_0:\sigma_1:\ldots:\sigma_{j-1}\kappa:$ $\kappa^{-1}\sigma_j:\ldots:\sigma_k]_n, j=1,2,\ldots, \text{ or } k.$

and

39. An equivalent $2^n \times 2^n$ k-stage bit-permuting network based on the j-th stage of a given $2^n \times 2^n$ k-stage bit-permuting network having the representation $[\sigma_0:\sigma_1:\sigma_2:\ldots:\sigma_{k-1}:\sigma_k]_n \text{ and based on a permutation } \kappa \text{ on integers from 1 to n that }$ preserves n, the equivalent network comprising

an input exchange $\sigma_0 \kappa$ if j=1, or an input exchange σ_0 if j=2,3,...,k, an output exchange $\kappa^{-1}\sigma_k$ if j=k, or an output exchange σ_k if j=1,2,...,k-1,

 $\label{eq:sigma_j} \text{interstage exchanges } \sigma_1,\,\sigma_2,\,...,\,\sigma_{j-1}\kappa,\,\kappa^{-1}\sigma_j,\,..,\,\sigma_{k-1} \text{ if } j=2,\,...,\,\text{or } k-1,\,\text{or} \\$ $\text{interstage exchanges } \kappa^{-1}\sigma_1,\,\sigma_2,\,...,\,\sigma_j\,,\,...,\,\sigma_{k-1} \text{ if } j=1,\,\text{or interstage exchanges}$

10 $\sigma_1, \sigma_2, ..., \sigma_j, ..., \sigma_{k-2}, \sigma_{k-1}\kappa$ if j = k.